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On correcting Errors of the Astronomical Circle by Opposite Readings. By T. R. Robinson, D. D. M. R. I. A. *Professor of Astronomy at Armagh.*

Read December 5, 1825.

WHILE engaged in making arrangements for obtaining the instruments which the munificence of the Primate has ordered for this Observatory,* it became an object of anxious enquiry whether the existing models of the astronomical circle were in any respects defective, and how far they might be susceptible of improvement. But I found it difficult to come to any conclusion on several important points; though I had the advantage of consulting men who are justly considered the brightest ornaments of Science, and of inspecting the noblest instruments in existence. Each observer, as he becomes habituated to his instrument, learns to correct, to avoid, or perhaps to overlook its defects, while he perceives the inaccuracies of other constructions in too strong a light: and the very close agreement between the results of all that are carefully used, shews that in truth the art of making and dividing instruments is nearly perfect. Yet the diversity of opinion which I found, made it desirable not to rely on authority alone, but if possible, to be guided by certain principles. The size of a circle is undetermined ; Mr. Bessel obtained excellent results with one of two feet ; and the largest which was ever constructed, the Dublin 8 feet circle, gives probably the most accurate conclusion : the lesser are less influenced by irregularities of temperature and by flexure ; the larger have their divisions more visible, and admit more powerful telescopes. This last consideration seems indeed to present a limit, for in the present state of the English glass-houses neither good crown or flint glass can easily be obtained, and a telescope longer than 5 feet can scarcely be got perfect. As to reading off divisions, I find that on the declination circle of our equatorial, (12 inches,) successive readings where the dot is good seldom differ more than $\frac{1}{3}$ of a second, and I found the same in a few trials of the great theodolite used in the Trigonometrical Survey : linear divisions are, I suppose, equally distinct.

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* A transit instrument and circle. The princely-minded founder of the Observatory, died before it was completed, and from circumstances which need not be detailed, those which *he* had ordered, were not obtained by the Observatory. It has at present a transit of 2 inches aperture and 3 feet focal length; but no instrument for polar distances except Troughton's equatorial, which, though excellent, is by no means capable of entering into the field with large meridian circles.

That construction which admits of being reversed in azimuth, seems at first sight to posssess many advantages over the mural circle ; the power of eliminating the error of collimation, and those which proceed from oblique illumination of the limb and inacuracy of the plumb-line or level, is of great importance ; nor is it less that many observations may be taken on the same day. On the other hand, mural circles are much steadier, and much more expeditious in their use ; and the happy invention of the collimator seems to remove all the objections that press upon them. This gives an invariable object of known altitude, by observing which, before or after the passage of a star, any disturbance of the microscopes is instantly detected,* and it seems as accurate as level, plumb-line, or even the bisection of the star itself, which is often no very precise operation. This therefore decided me to adopting a mural circle resembling those of Greenwich, except in the mode of supporting its axis, and the number of microscopes: it is not well adapted to act as a transit, but when there are two observers this is unnecessary; and the removal of half the axis gives great facility to observing by reflection when that is thought necessary, or of using the vertical collimator.* My chief embarrassment was in deciding on the number of microscopes, and here I was obliged to determine for myself upon the investigations, which I have the honour of laying before the Academy.

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* I find the Collimator a singularly useful appendage to the equatorial ; its superiority to the level is quite striking. A small *repeating* equatorial with its polar axis formed into a telescope,

Before the invention of circular instruments, we may wonder how observations could be made with the least pretensions to accuracy. Errors of division, accumulating through the whole limb of a quadrant, and unchecked by any accurate determination of the total arc, along with defects of the centre work, must have been present in those instances when certain Italian and German astronomers were found to be mistaken in their latitude 20 or 30 seconds, and from which a celebrated astronomer was led to conclude that they could never have made the observations which they reported. Ramsden, though not the first who constructed an entire circle, was the first who applied to it several readings, which, with his invention of the micrometer microscope, constitute an æra in astronomy. The circle of Palermo, in common with the others, executed by their great artist,

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and viewing an *inclined* collimator, would probably be the best portable instrument that has been devised.

* If the sun were observed by reflection from a surface of ink, it would obviate all heating of of the instrument. As to the vertical collimator, I do not know that it has been tried; by theory its stability is to that of the common one of equal weight as 1 : π. Ten grains placed at the extremity of my collimator, incline it 2.″

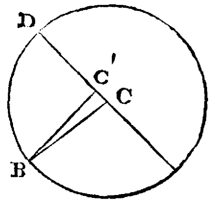
has two microscopes; so also those made of a portable size by Messrs. Cary and Troughton; the circles of Dr. Brinkley and Professor Gauss have 3, 90° asunder; Mr. Troughton makes his reflecting circles with 3 equidistant verniers, Mr. Reichenbach's instruments have, I believe, four, and the Greenwich circles have six microscopes. Among these influential names, that of Mr. Troughton will weigh the most with all who feel the high perfection to which he has carried the art of which he is the pride and ornament. He evidently considers multiples of 3 preferable to those of two. Mr. Pond also speaks highly of 3, though from circumstances probably connected with the difficulty of illumination, he uses but two when the utmost precision is not required.

Since the publication of Mr. Troughton's method of graduation, original errors of division are scarcely to be feared, and those which we have to consider arise from excentricity, from expansion of the materials of the instrument, and from the flexure by its own weight.

1. Excentricity is of two kinds which may be called variable and fixed; the latter existing when the line connecting the centres of the pivots on which the axle turns does not pass through the centre of its graduation, the former caused by a defect of the figure of the pivot. As the deviation from a circular figure is always minute, we may suppose the section of the pivot to be an ellipse of small excentricity: supposing this to roll in a rectangular Y, as is usual in astronomical instruments, it is easily shewn that its centre describes round the angle of the Y a circular arc whose radius is $r \times \sqrt{2 - e^2}$, and its sine $\frac{1}{4} e^2 \times \sin. 2\,\theta$, r being the greatest semidiameter of the pivot, θ the angle made by it with the vertical. Hence we see that the horizontality of an axis supported by such a pivot is not changed during its revolution, but that its extremity moves in azimuth through

the space 0. 35 $r \times e.^2$ sin. 2 θ, In consequence of this the circle revolves round its vertical diameter, and the deviation of its telescope from the meridian at an altitude α is as cos. $\alpha \times$ sin. $(2\,\alpha - \alpha')$, α' being the altitude at which the deviation in azimuth vanishes. The same reasoning applies to the transit instrument, and the errors arising from this cause may be easily ascertained and corrected. Besides this, the centre of the circle is also transported horizontally through a space bearing a constant ratio to the motion of the pivot; and this constitutes what I have called variable excentricity, different in no respect as to its influence on the measurement of angles from the permanent kind.

Fig. 1.



To determine what this is, let DB be an excentric circle, whose divisions are read by a microscope at B; while the circle is turned through the angle DC′B the arc DB passes before B, corresponding to DCB, less than the true angle DC′B by C′BC. Now sin. B : sin. CC′B : : CC′ : CB; or putting CC′ ÷ CB $= e$, DC′B $= \theta$, arc DB $=$ R and the angle B $= z$, we have

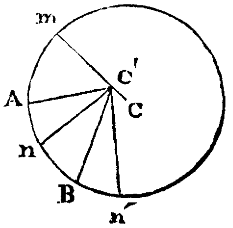
$$\theta = R + z, \quad \sin. z = e \sin. (R + z)$$

If e be so small that its powers above the first may be neglected, the second equation becomes

$$z = e. \sin. R.$$

Now let there be applied several microsscopes distant from each other by arcs $= a$.

Fig. 2.



In fig. 2. Let Cm be the radius passing through the centre of rotation, and C′n the line drawn from this through zero of the divisions; let B be the n^{th} microscope, A the first, then An is the arc corresponding to AC′n or θ, at A, Bn' at B. Both extremities of these arcs are effected by excentricity, therefore we must apply to them the difference of the values of z corresponding to those extremities, or for B,

$$\theta = R_n + e\,\{\sin.\,(mn'B) - \sin.\,(mn')\}$$

but

$$\text{arc.}\ mn'B = a.(n-1) - \text{arc.}\ mA,$$
$$\text{arc.}\ mn' = a.(n-1) - (\text{arc.}\ mA + R_n).$$

Let ϱ be the angular distance of Cm from zero, then

$$\text{arc}\ Am = \varrho - R,$$

and as $R = R_n$ nearly,

$$\theta = R_n + e\{\sin.\,(a.(n-1)+R-\varrho) - \sin.(a.(n-1)-\varrho)\}$$

At the other microscopes the coefficient of a is diminished by successive units, and therefore making it $= n$ and integrating, we have the sum or

$$n\theta = R + R_{,}\ldots\ldots + R_n + 2e\sin.\,\frac{R}{2} \times \Sigma.\left\{\cos.\,\left(na + \frac{R}{2} - \varrho\right)\right\}$$

The integral is

$$= \frac{\sin.\left(\frac{R}{2} - \varrho + \frac{2n-1}{2}\,a.\right)}{2\sin.\,\frac{1}{2}a} - C.$$

Determining the constant, so that the expression may vanish when $n = 0$, this becomes

$$\Sigma = \cos.\left(\frac{R}{2} - \varrho + \frac{n-1}{2}\,a.\right) \times \frac{\sin.\,\frac{na}{2}}{\sin.\,\frac{a}{2}}.$$

This vanishes if $na = 2\pi$, and then we have

$$\theta = \frac{1}{n}\{R + R_{,,}\ldots + R_n\}$$

or excentricity disappears from the mean of n microscopes when their distance from each other is the quotient of the circumference by their number.

Had z varied as any algebraic function of the sine or cosine of R, it must have similarly disappeared from the mean; for it could be expressed in constants and sines or cosines of the multiples of R, the constants disappear in taking the difference of the arcs $mn'B$ and mn', and the rest after integration assume the above form in which the part which is not proportional to θ is multiplied by a factor of the form

$$\frac{\sin.\left(\frac{s}{2}\times na.\right)}{\sin.\left(\frac{s}{2}\,a.\right)}=\frac{\sin.\,s\pi}{\sin.\frac{s}{n}.\,\pi}.$$

It therefore vanishes except when $s \div n$ is an integer, being then $\frac{0}{0}$, and having the value $\pm n$. Two microscopes therefore correct for excentricity or any other error varying according to the same law, as also for all which are as odd powers of the sine or cosine; three are still better, failing only where the number expressing the order of the error is divisible by three.

In general any error which is reducible to a series of these functions, or rather to the difference of two such, can be corrected by a certain number of equidistant readings. (*a*)

In Dr. Brinkley's circle the excentricity is not totally corrected, for $a = \frac{1}{2}\pi$, and $n = 3$, one microscope being wanting to complete the circle. The uncorrected error is

$$-\frac{2}{3}\,e.\,\sin.\left(\frac{R}{2}\right).\ \sin\ \left(\frac{R}{2}-\varrho\right)$$

which is a maximum when the altitude observed $= \varrho$, and becomes then

$$\frac{2}{3}\,e.\,\sin.^2\left(\tfrac{1}{2}\varrho\right).$$

But there does not seem to exist any excentricity in this instrument.

It may be remarked that no error is committed by assuming $= ze$.

sin. R, for developing the equation

$$\sin. z = e. \sin. (R + z)$$

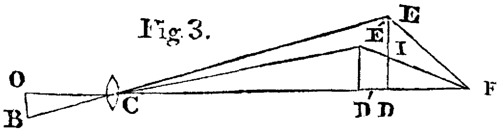
by Lagrange's well known theorem,

$$z = e. \sin. R + \tfrac{1}{2}e^2. \sin. 2R + \tfrac{1}{3}e^3. \sin. 3R, \ \&c.$$

and the entire expression comes within the application of Par. (a)

2. The excentricity of an instrument also produces an error of a different description, when the graduation is placed on the edge, as in the Greenwich circles, instead of the plane as is usual; for it alters the run of the microscopes. In fig. 3. let OC be the distance of the limb from the object glass of a microscope when correctly adjusted: let the image be then formed at ED, where the micrometer is placed whose screw measures ED the image of five minutes by five revolutions. If OC be increased by a quantity $= h$, the image is formed at E′D′, and when projected on the plane of the micrometer covers the space ID, for it is viewed by an eye-piece equivalent to a lens of the focal length FD placed at F. The difference between ID and ED is the error of the micrometer in 5′ which decreases from that to zero. Let DE = I, D′E′ = I′, DI = I″, OC = D, FD = f, and the focal length of the object glass = F, we have

Fig. 3.



$$CD - CD' = k = \frac{F^2 h}{(D - F).(D - F + h)}$$

$$I' = \frac{I.(D - F)}{D - F + h}$$

$$I'' = \frac{I' \times f}{f + k} = \frac{I.(D-F)^2.f}{f.(D-F)^2 + h.(f.(D-F) + F^2)}$$

$$\frac{I-I''}{I}=\frac{h.\left\{\frac{1}{D-F}+\frac{F^2}{f.(D-F)^2}\right\}}{1+h.\left\{\frac{1}{D-F}+\frac{F^2}{f.(D-F)^2}\right\}}$$

The quantity h is the effect of excentricity in the direction of the radius, or at the nth microscope.

$$h=re.\cos.\{R-\varrho+(n-1)\,a\}$$

substituting which in the above expression, and integrating, we should obtain the mean error of the n readings, if the Σ of such a function of n could be assigned in finite terms. Developing in powers of the cosine the value of $I-I''\div I$, it takes the form

$$c\cos.(na+b)-c^2.\cos.^2+\&c.$$

The odd powers disappear in n readings, but the even do not, for each of them contains a constant, and as in this case the error affects only one extremity of the arcs measured, they are not removed by subtraction. There remains in the mean

$$-\left\{\left(\frac{c}{2}\right)^2\cdot\frac{2}{1}+\left(\frac{c}{2}\right)^4\cdot\frac{3}{1}\cdot\frac{4}{2}+\&c.\right\}$$

where the general term is $\left(\frac{c}{2}\right)^{2s}\times\left(\frac{s+1}{1}\cdot\cdot\frac{s+2}{2}\cdots\frac{s+s}{s}.\right)$

With microscopes similar to those of my equatorial, which have $F=2$ and $D=3$ inchess, the first of those terms is $4081''\times h^2$ at its maximum. With a single microscope the total error is about $1565h''$, or a fourth of what the same value of h would produce as excentricity in a five foot circle.

In circles divided on the *plane*, variable exentricity produces the same error, though in a less degree, as also any deviation of the limb from its plane.

3. But a much more serious effect of the same kind is produced by the expansion of the circle from change of temperature; it is

indeed a real objection against this mode of placing the division, and almost overbalances the great stability, which is attained by fixing the microscopes to the pier. The expansion of the radii or arms of the circle brings the limb nearer to the object glass of the microscope, and that of the limb increases the magnitude of the interval between two divisions. The distance of the object-glass from the micrometer also increases, but this is nearly counteracted by the expansion of the micrometer screw.

Referring to the formulæ of the last section we have, neglecting the higher powers of h, and remarking that I becomes, by the expansion of the limb $I\ \{1+\varepsilon\tau\}$ where ε is the expansibility of brass for one degree, and τ the excess of temperature above that at which the microscopes were adjusted

$$\frac{I-I''}{I} = h\left\{\frac{1}{D-F} + \frac{F^2}{f(D-F^2}\right\} - \varepsilon\tau$$

but $h = -r\,\varepsilon\,\tau$, therefore

$$\frac{I-I''}{I} = \frac{\text{error in seconds}}{300} = -\varepsilon\tau\left\{\frac{r}{D-F} + \frac{rF^2}{f(D-F^2} + 1\right\}$$

a quantity much greater than in the former case, for in a five feet circle it is $= 0''47 \times \tau$; it does not receive the least compensation by increasing the number of readings, and there appears no remedy but that of registering the run of the microscopes at each observation, or until a table of its amount at different temperatures can be formed. If indeed they were attached to a circular frame similar and concentric to the circle itself, this error would be corrected, and with it probably those which are noticed in the following sections: the microscopes would be less stable, but it would be easy to apply a level or telescope to keep watch on them.

4. The unequal expansion of the arms of the circle deforms its figure, and that of the limb disturbs the equality of its divisions;

but here the greatest masters of analysis can scarcely see their way. Mr. Poisson has given in the Comm. des Temps. for 1826 formulæ for the distribution of heat through a metallic ring, and endeavoured to accomodate them to the case of an astronomical circle ; but if I may be permitted to say so, with doubtful success. I cannot refer to his original memoirs, but he seems to have overlooked the cooling effect of the air, as his co-efficient h in the expression of the fixed temperature seems to depend on the magnitude of the circle and its radiating power. Now instruments, such as are constructed in England, are made extremely light, so that every part of them can assume the temperature of the air in contact with it in a very few minutes; those however constructed by Reichenback, and the artists who imitate him, are exceedingly massive. If then we assume that the temperature of the air determines that of the circle with great rapidity, the former is so unsteady where the least agitation or current exists, that any reasoning which supposes regularity in the latter's distribution must be defective. That I do not exaggerate the rapidity of these changes of instruments, will be acknowledged by those who have observed the microscopes of a circle placed near an aperture which admits a current of air. But on this supposition there are two cases worth notice, because they come at once within the scope of Par. (a) : the first where the temperature of the air and therefore of the instrument increases uniformly with the height ; the second where it is any a function of the height alone. In the first Let T be the temperature of the centre, and $T + \tau$ that at the highest point of the circle, then that at the point whose angular distance from the lowest point $= \varphi$ is $T - \tau \cos \varphi$. If the arms of the circle were equally heated, they would expand equally, and the figure of the instrument suffer no change; but their lengths are unequal, being as the mean temperature of each arm is $T - \frac{1}{2} \tau \cos \varphi$

$$= \varepsilon\tau.(T - \tfrac{1}{2}\tau \cos.\ \varphi),$$

or taking T as the standard temperature, and putting $\varrho =$ the length of that arm which is horizontal, the change of each radius from ϱ is $= -\frac{1}{2}\,\varrho\varepsilon\tau.\ \cos.\ \varphi$, and the equation of the deformed circle becomes

$$z = \varrho.(1 - \tfrac{1}{2}\varepsilon\tau.\ \cos.\ \varphi.)$$

But this equation holds only for the extremities of the arms; which however from their superior strength command the limb: they also suffer a little flexure, as the arcs which connect them acquire by expansion a greater length than belongs to the portion of the above curve intercepted between the arms. Rectifying it by the usual method,

$$s = \varrho.\sqrt{1+\tfrac{1}{4}\varepsilon^2\tau^2.}\left\{\begin{array}{l} \varphi.(1 - \frac{1}{16}\,\frac{\varepsilon^2\tau^2}{(1+\frac{1}{4}\varepsilon^2\tau^2)^2} - \&c. \\ -\sin.\ \varphi.\ \left(\frac{\frac{1}{2}\varepsilon\tau}{1+\frac{1}{4}\varepsilon^2\tau^2} + \frac{3}{64}\,\frac{\varepsilon^2\tau^2}{(1+\frac{1}{4}\varepsilon^2\tau^2)^2} + \&c.\right. \\ -\sin.\ 2\varphi.\left(\frac{1}{32}\,\frac{\varepsilon^2\tau^2}{1+\frac{1}{4}\varepsilon^2\tau^2} + \&c.\right. \end{array}\right\}$$

Again the temperature of a differential portion of the arc is $(T - \tau.\ \cos.\ \varphi)$; while the arc increases by $d\varphi$, the sum of the temperatures of all its parts increases by this quantity, and therefore the mean temperature of the arc is

$$\frac{\int.\,(T - \tau\ \cos.\ \varphi).d\varphi}{\int.\, d\varphi} = T - \frac{\tau.\ \sin.\ \varphi}{\varphi},$$

which needs no constant. Therefore the mean expansion of the arc α intercepted between two arms is $-\ \varrho\varepsilon.\ \tau.(2\ \cos.\ (\varphi - \frac{1}{2}\alpha)\ \sin.\ \frac{1}{2}\alpha$, while the intercept of s between them is less than its original bulk by

$$\varrho.\sqrt{1+.\tfrac{1}{4}\varepsilon^2\tau^2.}\left\{\ \alpha.\left(\frac{1}{16}\,\frac{\varepsilon^2\tau^2}{(1+\frac{1}{4}\varepsilon^2\tau^2)^2},\ \&c. + \cos.\ (\varphi - \tfrac{1}{2}\alpha)\ \sin.\ \frac{\alpha}{2}\left(\frac{\varepsilon\tau}{1+\frac{1}{4}\varepsilon^2\tau^2} + \&c.\right.\right.\right.$$

little more than half the other. Therefore each arc must bend a little, and the difference of the forces required to produce this change of curvature in two adjoining arcs tend to bend the arm, thus producing a differential change in φ which is of no consequence in this enquiry. If tbe limb on the other hand should be so strong in the direction of the radius as to be nearly inflexible, it will command the arm, and in this case

$$s' = \varrho.(\varphi - \epsilon\tau.\ \sin.\ \varphi).$$

The length of the arc s is measured by the divisions of the instrument, which are no longer a scale of equal parts; let R be the number of them corresponding to the arc s, the length of that number at the temperature T would become $R + \epsilon\tau.\ \sin.\ \varphi$ as $R = \varphi$, nearly. Now it is evident that with the equidistant microscopes in either of these cases the terms affected with sines must disappear, and we shall have

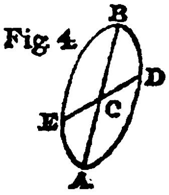
$$R + R_{,} \ldots R_{n} = n.b.t.$$

In practice I suppose the first most likely to occur.

These expressions coincide with the equation of a focal ellipse having the greater segment downwards, which accords with the determination of Mr. Poisson in the memoir already referred to.

If the temperature be merely uniform at a given height, it is evident that the distorted circle must remain symmetrical to its vertical diameter, which will cut its divisions in the same points as if it were undisturbed.

Fig. 4.



Let $ACD = \varphi$, we have, supposing arc $DA = \varphi + f\,\varphi$ where f denotes the function of the angle, which expresses the difference between the angle and its reading

$$f.\,2\,\pi = 0, \quad f.\,\pi = 0.$$

and as the arc EA + arc $EBDA = 2\,\varpi$,

$$f.\,\pi + \varphi + f.\,\pi - \varphi = 0.$$

Developing this last in powers of φ there results, as $f\,\pi = 0$.

$$0 = f^{\text{II}} \times \frac{\varphi^2}{1.2} + \frac{f^{\text{IV}} \times \varphi^4}{1.2.3.4.} + \&\text{c.}$$

or the even differential co-efficients must all vanish on the supposition $\varphi = \pi$. These conditions are satisfied by supposing

$$f\,\varphi = A \sin.\varphi + B \sin.2\,\varphi + \&\text{c.}$$

But any function of this form vanishes in n readings.

It is however to be feared, that even this degree of regularity seldom obtains, from the unequal expansibility of the different pieces of the instrument, which evidently cannot be reduced to any law: still there must be an approximation to it, and therefore to the correction of the error.

5. It has become fashionable to talk of the flexure of instruments since Bessel and Zach noticed its operation in their circles; and indeed it is a considerable temptation to admit its existence, that astronomers of acknowledged character find discordances for which they cannot otherwise account; but it is too much like the occult qualities of the Peripatetics to be freely admitted in these days. In quadrants it certainly was often a cause of great inaccuracy, producing permanent change of figure, but in circles it must occur in a far less degree, and be corrected by opposite readings. In the circles constructed by the German artists, where the telescope is supported in the middle only, projecting far beyond the limb, it is not surprising that its extremities should be unequally bent, notwithstanding the counterpoises that are applied to the tube.* But in those of this country, a single glance at the framing is sufficient to shew that the telescope cannot sensibly bend, much less the circle,

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* Zach once found a flexure in his telescope of three minutes, when the counterpoises were removed.

and the reasoning in the last paragraph shews that whatever differential disturbances may be produced, must be compensated by a number of readings, for the disturbing forces are manifestly functions of the angular distance from the vertical diameter.

This is perfectly consistent with experiment, for the largest circle in the world appears to have the same error of collimation from the horizon to the zenith, which is highly *improbable* on any other supposition. A rigorous determination of the shape which an instrument would assume by its own weight is extremely difficult; the necessity of considering the *transverse* elasticity of the arms and circumferences (or that which resists flexure) leads to an unmanageable equation of mixed differences; but by neglecting it as small in comparison of the forces which resist longitudinal extension or compression, perhaps the following rude method may determine with sufficient accuracy, a certain number of points in the deformed circle.

Let the circle be considered a polygon of as many sides as it has arms: part of the weight of each arm is borne by the centre, and the remainder $=W$ acts at its extremity, *pulling* the sides of the polygon that are above, and *pressing* those that are below it. If the material were inextensible, the first alone would happen, if incompressible, the second; both take place and beginning at opposite extremities of the vertical diameter, follow the same law, with opposite signs. Their sum therefore gives the actual tension or compression of each side, and compounding two adjacent, we get the force exerted on an arm in the direction of its length.

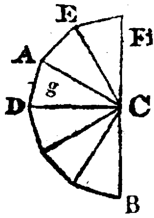

Fig. 5.

Let DA, fig. 5, be the m^{th} side, whatever force pulls AD, the same is communicated to AE increased by the action of W + half the weight of AD = W, supposed to be applied at its centre of gravity g. These therefore are the increase of the tension, while ∠ ACB = ma increases by ∠ ACE = a, or

$$\Delta t_m^{\bullet} = \frac{W.\sin.(ma). + W.\sin.\left(\frac{2m-1}{2}.a.\right)}{\cos.\frac{1}{2}a}$$

Integrating and determining the constant so that the whole may vanish when $m = 0$.

$$t_m = \frac{2\sin.\left(\frac{ma}{2}\right)}{\sin.a}\left\{W.\sin.\left(\frac{m-1}{2}.a.\right) + W.\sin.\left(\frac{m-2}{2}.a.\right)\right\}$$

If now we write for ma, $\pi - (m-1)\alpha$, we get the compression of the m^{th} side, and their difference is the actual force which affects it, or

$$F_m = 2\cos.\left(\frac{2m-1a}{2}\right).\left\{\frac{W+w.\cos.\frac{1}{2}a}{\sin.a}\right\}$$

Hence

$$F_{m+1} = 2\cos.\left(\frac{2m+1a}{2}\right).\left\{\frac{W+w.\cos.\frac{1}{2}a}{\sin.a}\right\}$$

and the resultant of these in the direction of the arm is

$$2\cos.(ma).(W+w.\cos.\tfrac{1}{2}a.$$

The arm will lengthen or contract a little by the application of this force according as it is positive or negative, and in proportion to it; its variation therefore is $\iota'.\cos.(ma)$, and the equation of the new shape of the circle is

$$z = r.(1+\iota'.\cos.\varphi)$$

a focal ellipse, but in an inverted position with respect to that resulting from the expansion of the circle.

The tension at any point is $-\frac{\epsilon'. \cos. \phi}{\sin. a}$, and therefore its effect in increasing the divisions is as in the last section $-\frac{\epsilon. \sin. \phi}{\sin. a}$: it may also be remarked that in this case the force required to change the curvature of the arc s does not interfere with the above equation only changing the value of ϵ'.

Even should one of the arms be stronger than the rest, yet the change of figure which this produces must be propagated through the others till it reaches the opposite point of the circle, and its effect at each arm depending on the curvature there, can evidently be expressed in terms of ϕ, and therefore this as well as the other errors produced by flexure, vanish on the supposition so often referred to of $a = \frac{2\pi}{n}$.

In the course of this investigation we have seen that generally all errors which have reference to the position of the circle are corrected in this way, except the effect of expansion on the microscopes. It may also be remarked that two microscopes must be sufficient in most instances, for the co-efficients e, ϵ and ϵ' are so small that their second powers can have no sensible effect; yet we can conceive cases where they might fail. If the instrument were an ellipse concentric with the original circle, as in it the difference between s and ϕ is as sin. 2 ϕ, three would be required, and such an occurrence is by no means impossible. Circles are generally turned on their pivots, and of course any imperfection in the latter is transferred to the former. In this process, any jarring or shake has a tendency to propagate itself round the circumference in equidistant waves, and the number of these gives that multiple of ϕ on whose sine or cosine the error to be corrected depends. When this is divisible by the number of microscopes, the error is not corrected, and therefore Mr. Troughton's preference of three readings to two is fully confirmed. There seems no great advantage to be expected from encreasing the number beyond this, and the difficulty of using and keeping them in order is considerable.

On the whole I think we may conclude that the construction of our circles is not as defective as some suppose; they correct their errors, and the only thing which is wanting to make them as perfect as the essential properties of matter allow is some effectual check on the stability of the microscopes. It is impossible to depend on any fixture of them however firm, and we want some means either of eliminating their deviations from our result, or of measuring and allowing for them. The observations made at Greenwich with *fixed* telescopes, where all the resources of art and science were employed, shew strikingly how impossible it is to fix any thing; and indeed in all things connected with *instruments*, we are not warranted to suppose ought that is susceptible of proof. This is not the least important lesson which we derive from the labours of our predecessors in science.

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